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NOTES ON PROPELLER DESIGN - IV:

General Proceeding in Design.

By Max M. Munk.

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Summary.

The choice of the number of revolutions and of the diameter, the distribution of thrust, and the values of the constants in the aerodynamical equations of the propeller are discussed.

The exact design of a propeller must be preceded by approximate computations, leading to the general layout, which in turn must be followed by an analytical examination of the propeller, obtained under several conditions of flight. In the previous notes of this series I have discussed these three steps, including the determination of the distribution of thrust, at length. It will be useful to summarize the procedure briefly in this note and to discuss some general principles in connection with it. For the design of a propeller is a laborious undertaking, and the analysis of the finished design ought to confirm that it is correct. The analysis is not the proper method for studying the

effects of different assumptions for the layout. This can be done more shortly and more successfully by a general discussion.

The number of revolutions of the propeller is, in general, determined by the engine; but the problem often remains as to whether reduction gearing is advisable. Now the reduction gear always has a friction loss of 2% at least. What is more serious, it requires a great additional weight of the propeller and gear, it gives rise to increased stresses of the fuselage, and it involves additional complications and possibilities of dead stops. It is expensive too. These disadvantages cannot exactly be taken into numerical consideration. But it can be safely said, I think, that the reduction gear is inadvisable if the aerodynamical efficiency of the propeller is increased less than 5% by its application. There are designers who prefer even an aerodynamic loss of 10% to a reduction of the number of revolutions by gearing.

The question of the diameter and of the number of revolutions is not exclusively a question of efficiency. There are of course upper limits for the velocity of the blade tips. Besides, the design of the propeller is carried out, having in view one particular condition of flight, whereas the propeller is to be used under very variable conditions. It is desirable that (a) the efficiency be fairly high under all conditions and that (b) the absorbed horsepower at the same number of revolutions remain nearly constant. Only if these two requirements are fulfilled

at the same time, can the propeller develop the highest thrust horsepower. These two requirements lead now to a limitation of the tip velocity of the propeller blades. For this tip velocity determines the average velocity of the blade elements and hence the variation of their angles of attack. But it is this angle of attack which has to conform to the variation of the thrust, and hence the variation of the angle of attack determines the lift coefficient. A calculation shows that for constant density of the air the best lift coefficient corresponds to the usual tip velocity, high enough for a favorable ratio C_L/C_D. This appears if one applies the aerodynamical equations for the blade element. Hence the designer has to keep in mind that a change in the tip velocity has a noticeable effect either on the behavior of the propeller over a wide range of conditions or on its efficiency, and that this effect cannot be neutralized by minor changes in the design.

The question is intimately connected with that of the advisability of variable pitch. I have just mentioned the fact that for constant density a satisfactory propeller can be designed having constant pitch. Variable pitch is useful only for great altitudes. But then indeed it can greatly improve the performance of the propeller. The variation of the pitch enables the propeller to conform to an additional variation of the conditions, that is, the variation of the density of air.

Some designers of variable pitch propellers claim that the

center of pressure of the blade sections of their propellers does not travel. Whether this be true or not, it has nothing to do with the variability of the pitch. For the travel of the center of pressure is due to the change of the lift coefficient of the blade elements, which always takes place for different conditions of flight, because the lift coefficient is nearly proportional to the thrust, and the thrust changes. Hence the center of pressure does not travel if the blade section used is one without travel of center of pressure, whether the pitch be changeable or not.

Proceeding now to the general layout, I have shown that the diameter must satisfy the condition:

$$(1) \qquad D < \sqrt[3]{\frac{T}{nV\frac{\rho}{2}} \; C_L/C_D \; \frac{6}{\pi^2}}$$

where T denotes the thrust,

ρ the density of air,

C_L/C_D the lift/drag ratio of the blade section.

This equation gives too high a value for a small velocity of flight such as occurs during the start, for instance. C_L/C_D can be assumed to be 22. The diameter thus obtained is only a rough indication of the upper limit; the weight, the stresses and the structural point of view are not yet taken into account. In general the diameter is given by other considerations, and the following method is valid for any diameter, however determined.

After having made the decision as to the magnitude of the diameter, the required horsepower is to be estimated and to be compared with the horsepower delivered by the engine. The thrust horsepower may be written $N_o = T\, V$. The sum of the thrust horsepower and slip stream loss is then

$$N_o \frac{(1 + \sqrt{1 + C_p})}{2} \quad \text{where} \quad C_p = \frac{T}{D^2 \frac{\pi}{4} V^2 \frac{\rho}{2}}$$

The friction loss is $.033 \; N_o \frac{\pi Dn}{V}$. Hence the smallest brake horsepower possible is approximately

$$(2) \qquad N_o \left[\frac{1 + \sqrt{1 + C_p}}{2} + .033 \frac{nD\pi}{V}\right]$$

The factor 0.033 refers to average conditions and to a lift coefficient of the blade of about 0.4 to 0.8. For smaller lift coefficients the factor is greater, and for higher it may be smaller, say up to $C_L = 1.2$, and then greater for still higher lift coefficients. This depends on the blade section.

The next step is the determination of the number and breadth of the blades. First, the lift coefficient is to be assumed. The highest lift coefficient occurring ought to be about 0.80 to 1.10. It can be said that the lift coefficient is fairly proportional to the thrust. Its value to be chosen is therefore in the neighborhood of T/T_{max} where T is the thrust for which the propeller is designed and T_{max} the greatest thrust ever occurring for the propeller.

The lift coefficient being chosen, the product of the number of blades and the average breadth of each blade is approximately

$$(3) \qquad i\ t = \frac{6\ T}{C_L \frac{\rho}{2} n^2\ D^3 \pi^2}$$

This formula makes it possible to decide on the number of the blades and on their breadth.

The general layout of the propeller is thus finished and the design in detail can begin. First, the distribution of the thrust is to be decided upon. The coefficient of thrust is to be assumed, taking

$$(4) \qquad C_T = 1.1 \frac{T}{q\ D^2 \frac{\pi}{4}}$$

at 2/3 of the radius and

$$(5) \qquad C_T = 1.1 \frac{T}{q\ D^2 \frac{\pi}{4}} - \frac{2}{3}\ V/U\ \ C_D/C_L\ \ \frac{\pi\ n\ D}{V}$$

at the tip, where q is the dynamic pressure. For a very low velocity of flight, the last expression can be slightly decreased. This is, however, not yet the definite value of the thrust coefficients at these points. A diagram is to be drawn now, plotting the thrust coefficient against the radius. The two values just calculated are put in and connected by a straight line. Then the two ends of the curve so obtained are rounded off. At the outer end the rounding may begin, say 10% of the radius from the end, and the curve may end elliptically. At the inner end, the thrust

coefficient is zero over the hub, and near it, only a small density of thrust can be realized, say

$$\frac{1}{2}\left(\frac{\pi n r}{V}\right)^2$$

or even the same expression with a factor smaller than $\frac{1}{2}$, if there are only two blades and these narrow ones. This gives a limiting curve inside, which is to be rounded off where it intersects the first one.

One must now see whether the curve of thrust coefficient thus obtained gives the desired thrust. The radius is to be divided into a number of equal parts Δr, say 10 parts. For each part the average value of the thrust coefficient is to be taken from the diagram and is to be multiplied by the radius r. All these products are to be added, and the sum A so obtained is to be multiplied by $2\pi \Delta r \cdot V^2 \rho/2$, thus giving

(6) $$T_1 = A \quad 2\pi \quad \Delta r \quad V^2 \frac{\rho}{2}$$

This thrust will not agree exactly with the desired thrust T. Therefore all coordinates C_T are to be increased by the constant additional term $(T - T_1)/(.78\ D^2 q)$. Then they represent C_T for each blade element, and the section of each blade element and its inclination can be laid down.

For the choice of the blade sections the same rules are valid as for the choice of wing sections. The angle of attack for the desired lift coefficient can be calculated or it can be taken from a model test. In the last case the induced angle of attack

$\frac{C_L}{\pi}\frac{t}{b}$ where b/t is the aspect ratio of the wing model, is to be subtracted from the angle obtained during the test. The drag too is to be reduced to the drag for infinite aspect ratio, and an additional constant reduction of the drag coefficient making the minimum drag coefficient 0.02, will improve the result.

Let now δ be the difference of the angle ϵ of inclination of the blade element with respect to the propeller plane and the angle of attack β. This angle δ is to be determined by means of

$$(7)\qquad \tan\delta = V/U + \tan\delta\ (1 + V/U)\left\{1 + \frac{2\sin^2\delta}{C_T} - \sqrt{\left(1 + \frac{2\sin^2\delta}{C_T}\right)^2 - 1}\right\}$$

where U denotes $2\pi r n$. This equation is to be used by substituting an approximate value of $\tan\delta = 1.1\ V/U$ in the right-hand term. Sometimes the proceeding is to be repeated by substituting the value of $\tan\delta$ thus obtained. When $\tan\delta$ is found, the angle of inclination is

$$(8)\qquad \epsilon = \delta + \beta$$

and the chord of the blade is obtained:

$$(9)\qquad t = 4\sin\delta\ \frac{\tan\delta - V/U}{1 + V/U}\ \frac{2\pi r}{i\ C_L}.$$

For the analysis the following equations are used:

$$(10)\qquad \tan\delta = \frac{\tan\epsilon\ (1 + V/U)\ \sin\delta + V/U\ \frac{4r}{i\ t}\ (\tan\epsilon\ \tan\delta + 1)}{(1 + V/U)\ \sin\delta + \frac{4r}{i\ t}\ (\tan\epsilon\ \tan\delta + 1)}$$

$$(11)\qquad C_L = 2\pi\ (\epsilon - \delta).$$ With some sections, C_L is slightly less.

(12) $$m = \frac{i\ t}{2\pi r} \quad \frac{(\epsilon - \delta)\pi}{2}$$

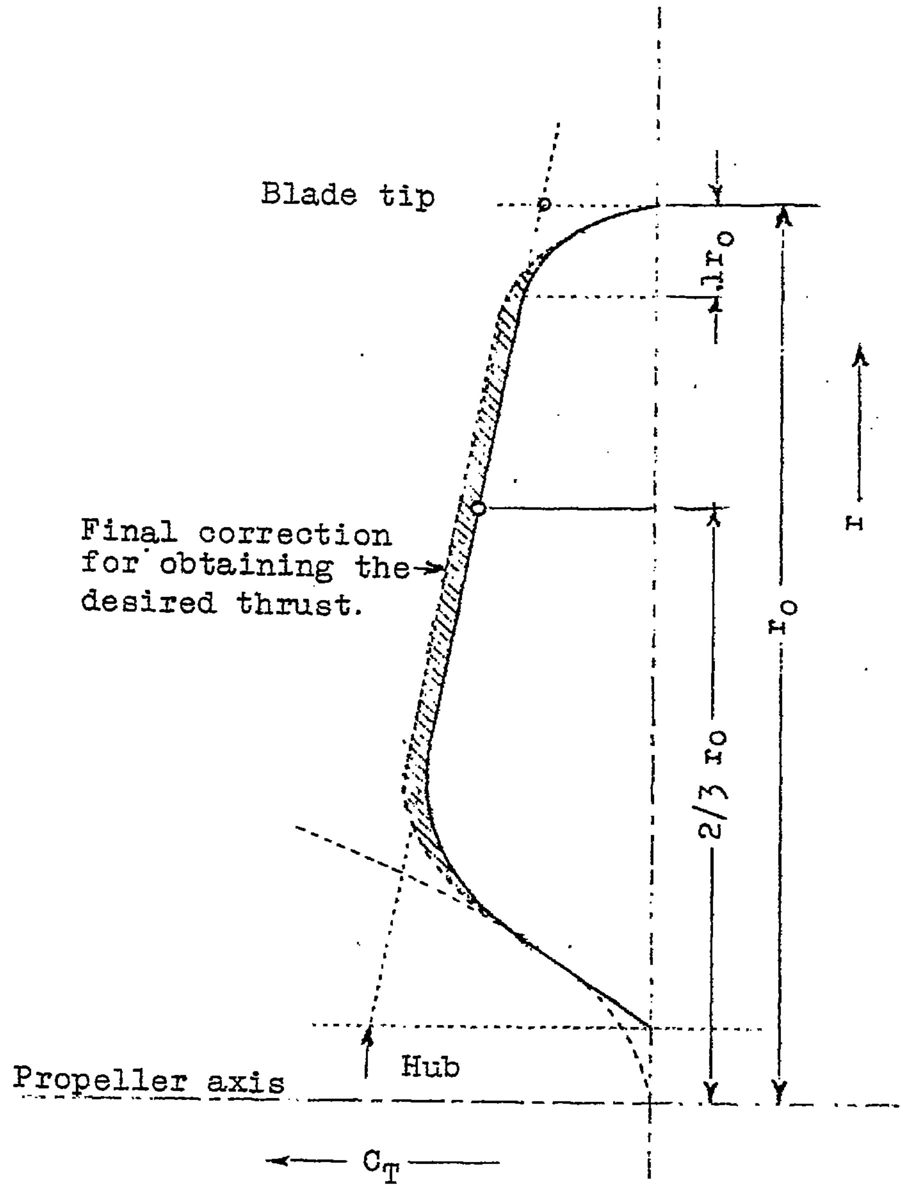
(13) $$C_t = \frac{4\ m \cos\delta}{1 - m \cos\delta/\sin^2\delta}$$

(14) $$C_Q = C_T\ (\tan\delta + C_D/C_L).$$

(15) $$T = \Sigma\ C_T\ 2\pi\ r\ \ \Delta r\ \ \ V^2\ \frac{\rho}{2}$$

(16) $$\text{Torque} = \Sigma\ C_Q\ 2\pi\ r^2\ \ \Delta r\ \ \ V^2\ \frac{\rho}{2}$$

Equation (10) is to be used instead of equation (7), assuming that $tg\delta$ lies between V/U and $\tan\epsilon$.


Blade tip
.1r o
Final correction
for obtaining the
desired thrust.
r
r o
2/3 r o
Hub
Propeller axis
C T